

## DESIGN AND BUILD A PROTOTYPE VOLTMETER CALIBRATOR

### Background:

The units of electric and magnetic quantities are all directly related to the mechanical units of distance, mass, and time. For example, electric charge,  $Q$  can be defined in terms of the force between two point charges (Coulomb's law)

$$F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} . \quad (1)$$

Here,  $\epsilon_0$  is a constant defined to be  $8.85 \times 10^{-12}$  Farads/meter. Electric field,  $E$ , is defined as force per unit charge exerted on a test charge,  $Q_2$ , so the electric field from a point charge,  $Q_1$ , is

$$E = \frac{Q_1}{4\pi\epsilon_0 r^2} . \quad (2)$$

Voltage is defined to be the potential energy per unit charge of a test charge,  $Q_2$ . The potential energy difference between two points is the integral of the force over a path between the points. Thus the voltage difference between two points at distance  $r_1$  and  $r_2$  from a point charge,  $Q_1$ , is obtained by integrating equation (2) between the two positions, yielding

$$\Delta V = -\frac{Q_1}{4\pi\epsilon_0 r_1} + \frac{Q_1}{4\pi\epsilon_0 r_2} . \quad (3)$$

### Objective:

In this experiment you will construct a device to measure the absolute value of a voltage through the measurement of a force. The actual measurements that you will make will be of mass, distance and time, but the result will be a measurement of an electrical potential in Volts. This can be accomplished with the procedure outlined below,

### Reference:

Halliday, Resnick, and Walker, 5th edition Chapter 25, sections 1 through 5.

### Description of the method:

Rather than attempt to simulate point charges to satisfy Coulomb's law, we will measure the force that results from applying a potential (voltage) between capacitor plates. That force is derived from Coulomb's law. Assume that we make two flat plates of radius  $R$ , and place them parallel to each other at a separation,  $d$ , such that  $d \ll R$ . Then we can, to an accuracy we can estimate, compute the field as if the plates were infinitely large. If a voltage is applied between the two plates, some charge,  $Q$ , will be removed from one plate and pushed onto the other. The plates, if originally neutral, will then each have a charge  $Q$  of opposite sign. Gauss' law tells us that the electric field between the plates is then given by

$$E = \frac{Q}{A\epsilon} , \quad (4)$$

where  $A$  is the area of each plate, and  $\epsilon$  is the dielectric permittivity of whatever material is between the plates. ( $\epsilon$  for air is close enough to that for vacuum,  $\epsilon_0$ , so that, to the accuracy of this experiment, the two can be treated as equal.)

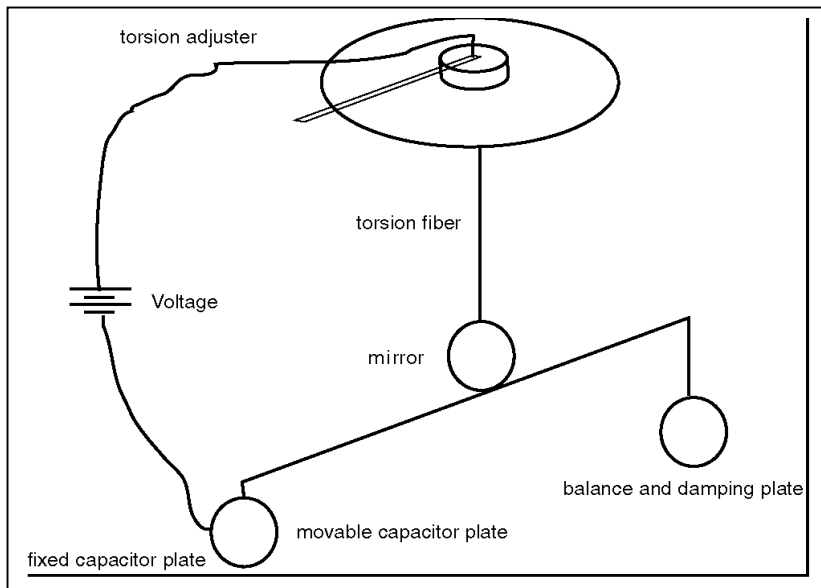
The voltage between the plates is given by the integral of  $E$  over the distance between the plates. Since  $E$  is constant we have

$$V = Ed = \frac{Qd}{A\epsilon} . \quad (5)$$

The force between the plates is equal to the charge on one plate times the  $E$  field due to the other plate (which is  $1/2 E_{\text{TOT}}$  by symmetry)

$$F = \frac{1}{2}EQ = \frac{1}{2} \frac{Q^2}{A\epsilon} = \frac{1}{2} \frac{A\epsilon_0}{d^2} V^2 . \quad (6)$$

Thus if we can measure the force  $F$ , the area  $A$ , and the distance  $d$ , we can determine the absolute value of the voltage applied to the plates.



A simple and accurate method to measure the force is to attach one of the plates to a torsion pendulum, while keeping the other plate fixed. The figure shows a capacitor plate electrically connected to the left arm of the torsion pendulum. (An identical plate is attached to the right arm to balance the pendulum and to damp the pendulum oscillations by placing it in water).

The torque resulting from the electrostatic force will be  $Fl$ , where  $l$  is the horizontal distance between the center of the capacitor plates and the torsion fiber. The angle through which the torsion pendulum will rotate from this torque is  $\theta = Fl/\kappa$  where  $\kappa$  is the torsion constant of the suspension (see text section 16-5). The value of  $\kappa$  can be determined by measuring the period of the pendulum, weighing the two plates and computing their moment of inertia,  $I$ . Then, using the expression for the period of a torsion pendulum to solve for  $\kappa$ , we have

$$T = 2\pi\sqrt{\frac{I}{\kappa}} \quad (7)$$

Measuring the angle of rotation directly would complicate the measurement and reduce its accuracy since the separation  $d$  between the capacitor plates would also vary with the angle. A better procedure is to hold the separation of the capacitor plates fixed as the voltage between them is increased. This can be done by twisting the top end of the fiber enough to exactly cancel the torque from the applied voltage. The angle of rotation of the top of the fiber is measured by the angle through which the torsion adjuster is rotated. In order to determine accurately that the pendulum remains in the position it had with no voltage applied, we will reflect a light beam

from a fixed source off of the mirror onto a screen. The torsion adjuster will then be twisted as the voltage on the capacitor plates is increased so that the image on the screen remains in its original position.

Notice that the torsion fiber and horizontal arm must be metallic in this apparatus so that the voltage difference between the capacitor plates can be applied without an additional wire placing a torque on the suspension.