Physics 4L Laboratory 4 (2026) - High-order circuits and resonators

4.1. Ladder network

This exercise addresses simultaneous measurements across multiple sites and serves to increase intuition about RC circuits. Build the ladder circuit in Figure 4.1 using R = 2.2 kΩ and C = 0.47 μF.

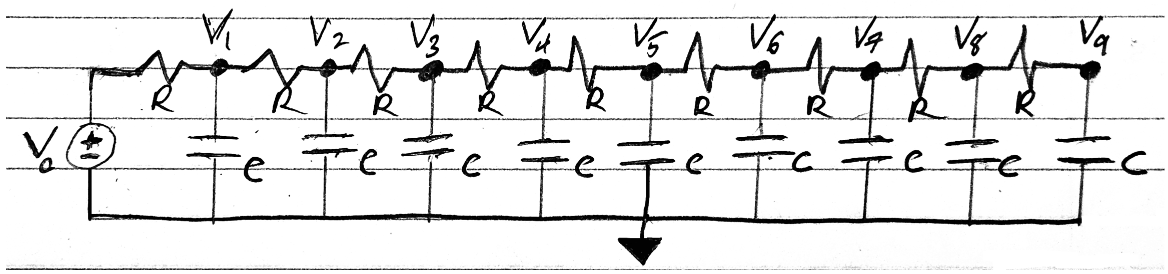

Figure 4.1: A RC ladder network that acts as a nine-pole low-pass filter.

Choose V_O to be pulse of 0.5 s in period and 250 ms in width (50 % duty cycle). Measure in four places: V_O, V_3; V_6, and V_9. The input Vo can be measured with a BNC cable from the waveform generator, but make sure to use scope probes for V_3, V_6, and V_9. Be sure to set each channel's probe attenuation appropriately.

Q1. Expand the region around the onset of the pulse. What differences do you see for the different locations? Take a SCREENSHOT and explain.

Q2. Now change to a very short pulse, i.e., an impulse, that is realized by changing the duty cycle of the pulse. What differences do you now see for the different locations? Take a SCREENSHOT and explain.

By the way, a ladder can model a discrete version of the diffusion equation, with $D \propto 1/RC$.

4.2. RL circuit

We pick up the second component with a time-varying I-V relation, the inductor, and use RL circuits to explore the interpretations of different inductor/resistor configurations.

Build the RL circuit, configured as a low-pass filter, shown in Figure 4.2:

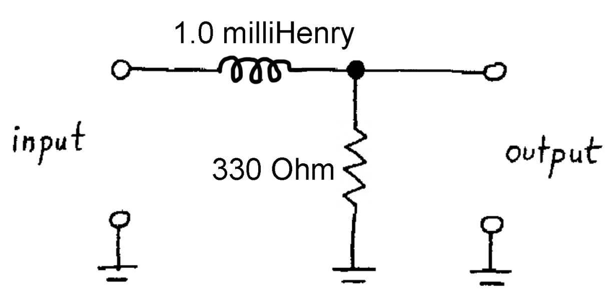

Figure 4.2: RL circuit as a low-pass filter.

For a step input of amplitude V_{in}, theoretical calculations for ideal components predict that the voltage should rise as $V_{out} = V_{in}[1 - exp(-t/\tau)]$ at the onset of a long pulse and decay at the offset of a long pulse as $V_{out} = V_{in} \cdot exp(-t/\tau)$, where $\tau \equiv L/R$ ($\cong 3\ \mu$s in this example).

Q3. Record the input wave and the output of the RL circuit as a SCREENSHOT from the oscilloscope. Measure the time constant τ.

Q4. Does the measured time constant equal the quotient L/R?

Recall that **R**s and **L**s have tolerances, so measure the exact values for your components. In this case, the inductor also has an internal resistance of about $\mathbf{r_L}$ - 31 Ω, or 10 % the value of the output resistor, so a more exact circuit is given by Figure 4.3

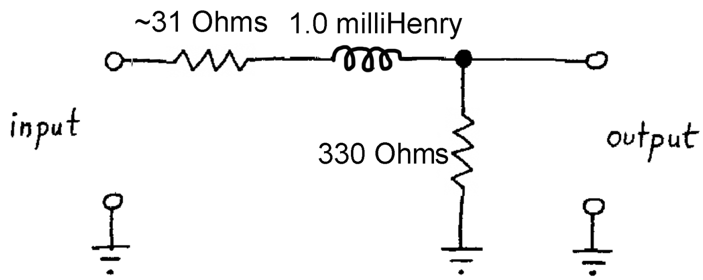

Figure 4.3: RL circuit as a low-pass filter showing lumped internal resistance of the inductor.

Q5. Make an improved estimate of the time constant by incorporating $\mathbf{r_L}$. Does the measured time constant equal the improved estimate?

Steady state voltage will be lower than the requested waveform generator voltage. This is because there is a Thevenin's resistance to the waveform generator. Similarly, the time constant will be shorter than estimated.

Q6. Estimate the Thevenin's resistance of the waveform generator and see if your estimate of the time-constant improves.

4.3. LCR circuit - step response

We use LCR circuits to explore the nature of oscillatory circuits. We can build a RLC resonator with either the capacitor and inductor in parallel or in series. While the ideal circuits are readily solved in both configurations, the presence of a non-negligible resistance in the inductor makes the mathematical solution for latter configuration much simpler. Thus we choose the serial circuit of figure 4.4, with the caveat that the load on the function generator will be severe when the drive frequency is near the resonant frequency and the total resistance "small". Thus we will have to correct for this - your first (?) experience with the reality of imperfect measurements.

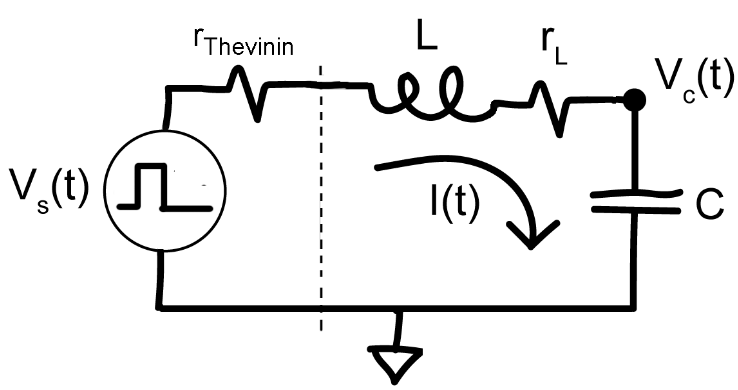

Figure 4.4 The serial RLC circuit.

We choose as components **L** = 15 mH (accompanied by $\mathbf{r_L} \approx 30$ Ω and $\mathbf{r_{Thevenin}}$ to be determined), **C** =33 nF. Check all the values with a meter!

Q7. What is the expected value of the free-induction decay frequency, $f_0 = \omega_0/2\pi$, if everything is ideal, i.e., $r_L = r_{Thevenin} = 0$? What is the expected value of f_0 using your measured value of r_L (ignoring $r_{Thevenen}$) and the formula from class notes?

We first consider the response of the RCL circuit to a step to study the undriven response. Add an adjustable resistance, denoted **r**, to the series LCR circuit using the resistor substitution box as shown in **Figure 4.5.**

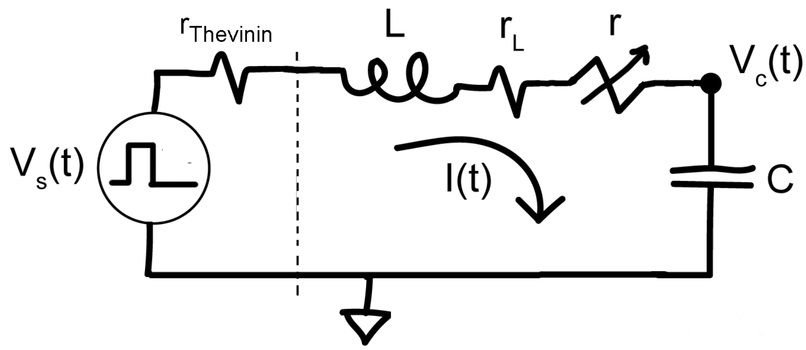

Figure 4.5 The serial RLC circuit with additional resistance **r**.

Use the pulse function of the waveform generator for the step $V_S(t)$. Set the frequency to be around 50 Hz and have the amplitude range from zero to 10 V. Splice in a load resistance using the decade resistor substitution box, as in Figure 4.5. Make sure the pulse is wide enough for the output of the circuit to reach it's fianl state; we will measure at the falling edge of the pulse.

Q8. Choose r = 0. Measure the response of the circuit at the falling edge of the pulse. Take a SCREENSHOT of $V_C(t)$ and $V_S(t)$ and explain.

Q9. What is the frequency of the circuit underdamped response? Does it match the calculated value?

Hint: You can measure the frequency very accurately using cursors at zero-crossings. Zoom in to the first and second zero crossing to do this frequency measurement. This will be very helpful as you slowly increase **r** and diminish the frequency.

Now slowly increase **r** in steps of 100 Ω. Carefully expand the amplitude and time-base to observe the off response.

Q10. Qualitatively, what do you see as r increases?

When the resistance satisfies critical damping, the overshoot will disappear.

Q11. Take a SCREENSHOT of the expanded region around the off response. Explain the nature of the critically damped response.

Q12. Does the resistance that you chose to achieve critical damping match the calculated value?

Continue to increase r.

Q13. Take a SCREENSHOT to demonstrate over-damping.