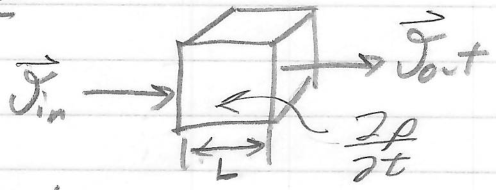
Phys 4L - Lecture 1

Two laws from E&M give rise to electrical circuit analysis

Conservation of charge

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

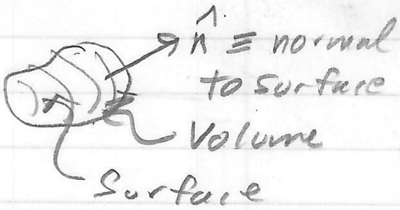


$\vec{J}$ is the current density with units of $Q/L^2T = Q/L^3 \; L/T$ = charge density x velocity

ρ is the charge density with units of Q/L^3

If charge is not created or distroyed, $\frac{\partial \rho}{\partial t} = 0$ and $\vec{\nabla} \cdot \vec{J} = 0$. But by the Divergence Theorem

$$\int_{Vol} \vec{\nabla} \cdot \vec{J} \, d^3r = \int_{Surf} \vec{J} \cdot \hat{n} \, d^2r$$

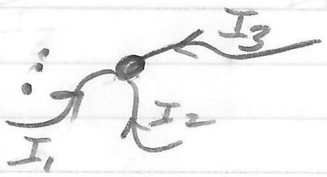


$$'' = I_{out} - I_{in}$$

where I is current with units Q/t. In general

$$\sum_i I_i = 0 \text{ at a node}$$

This is Kirchoff's Current Law.



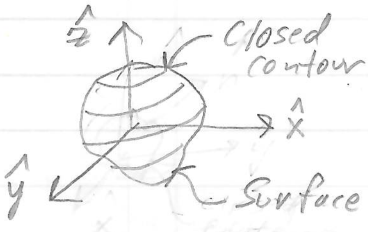
Faradays Law

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$\vec{E}$ and $\vec{B}$ are, respectively, the electric and magnetic fields.
If there is no $\vec{B}$-field or a constant $\vec{B}$-field, then $\vec{\nabla} \times \vec{E} = 0$. But by Stoke's theorem

$$\oint (\vec{\nabla} \times \vec{E}) \cdot \hat{n}\, d^2 r = \int_{\text{closed contour}} \vec{E} \cdot \hat{n}\, dr$$

($d^2 r$: open surface)



$$'' \doteq \int \vec{E} \cdot \hat{x}\, dx + \int \vec{E} \cdot \hat{y}\, dy + \int \vec{E} \cdot \hat{z}\, dz$$

$$\vec{E} \cdot \hat{x} \equiv E_x = -\frac{\partial}{\partial x} V_x \text{, etc.}$$

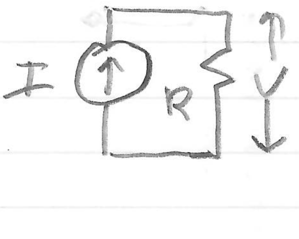
$$'' = -\int \frac{\partial V_x}{\partial x} dx - \int \frac{\partial V_y}{\partial y} dy - \int \frac{\partial V_z}{\partial z} dz$$

$$'' = -V_x - V_y - V_z$$

$\therefore \sum_i V_i = 0$ around a loop

This is Kirchoff's Voltage Law

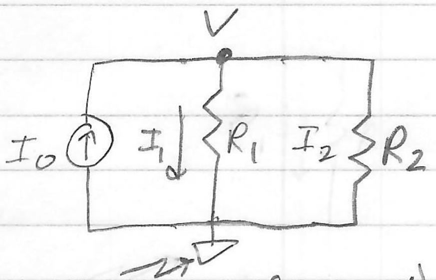
Ohm's Law



$V = IR$ ← Resistance

Examples

<u>Current division</u>



One node can be set to zero (ground)

Sum of currents flowing into node "V"

$$0 = -I_0 + V/R_1 + V/R_2$$

$$V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = I_0$$

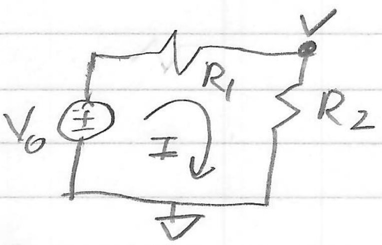
$$V = \frac{R_1 R_2}{R_1 + R_2} I_0$$

$$I_1 = V/R_1 = \frac{R_2}{R_1 + R_2}$$

$$I_2 = \frac{V}{R_2} = \frac{R_1}{R_1 + R_2}$$

} current divides based on conductance $G \equiv 1/R$

<u>Voltage divison</u>

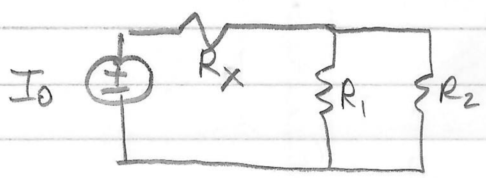


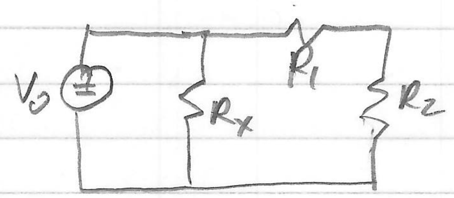
Sum of voltage drops around loop

$$\frac{V - V_0}{R_1} + \frac{V}{R_2} = 0$$

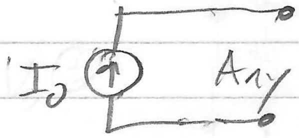
$$V = V_0 \frac{R_2}{R_1 + R_2}$$

<u>Irrelevant Resistors</u> "R_x" – Why?



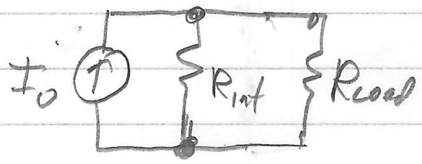


Ideal Sources vs Real Sources



Any voltage can develop in an ideal source

Max of $I_0 R_{int}$ voltage drop because of internal resistance



$$I_{Load} = \frac{R_{int}}{R_{int} + R_{Load}} I_0$$

$$\xrightarrow[R_{int} \to \infty]{} \frac{I_0}{1 + R_L/R_i} \simeq \left(1 - \frac{R_L}{R_i}\right) I_0$$

When $R_{Load} \to \infty$, no current flows

$R_{Load} \to 0$, no voltage drop occurs

Since Power $= IV$, there is an optimal R_L

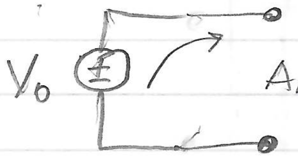
$$\text{Power} = I_L V_L = I_L^2 R_L = I_0^2 \frac{R_i^2 R_L}{(R_i + R_L)^2}$$

$$\frac{\partial \text{Power}}{\partial R_L} = I_0 R_i^2 \frac{(R_i^2 - R_L^2)}{(R_i + R_L)^2}$$

$= 0$ for $R_i = R_L$

Maximum Power in the Load $= \left(\frac{I_0}{2}\right)^2 R_L$

Equal Division of Current



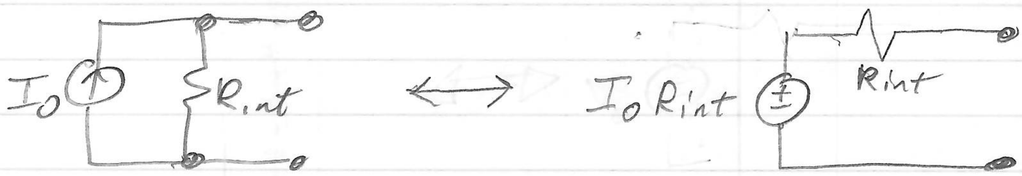
Any current will flow

Max of $\frac{V_0}{R_{int}}$ current because of internal resistance.

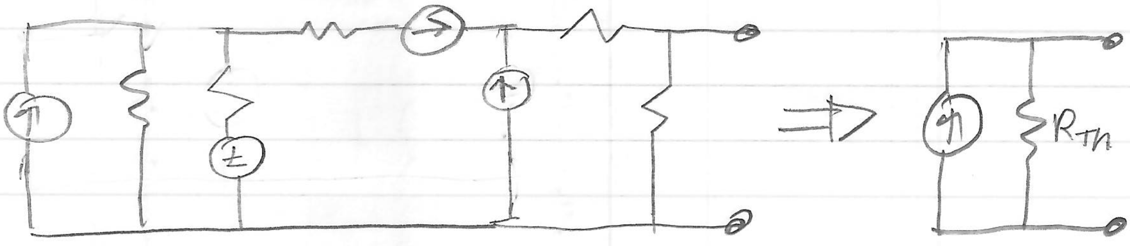
$$V_{Load} = V_0 \frac{R_{Load}}{R_{int} + R_{Load}}$$

$$\xrightarrow{R_{int} \to 0} V_0 \left(1 - \frac{R_{int}}{R_L}\right)$$

Equivalence of sources



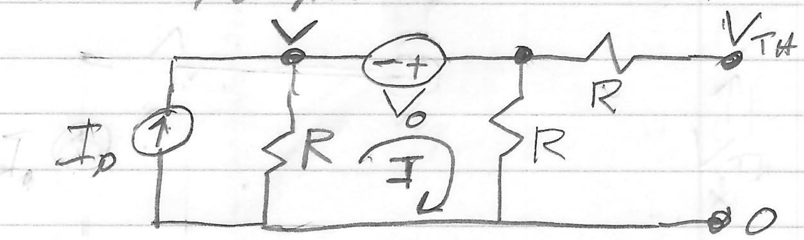
In general, any circuit of resistors, current sources, and voltage sources can be reduced to a "Thevenin" equivalent of one source and one resistor (R_{TH})



① Calculate open circuit voltage at output
OR
② Calculate short circuit current at outup

③ Calculate equivalent resistance with current sources as open circuits, voltage sources as short circuits

An example

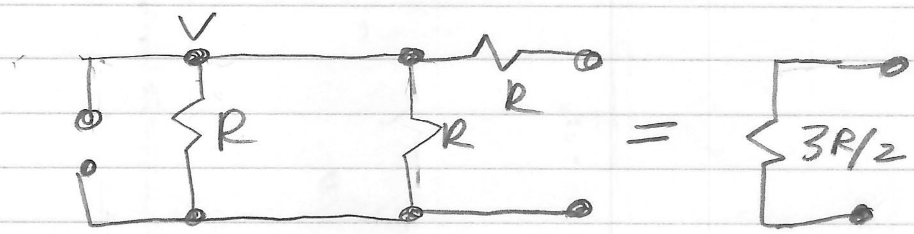


$$-I_0 + \frac{V}{R} + I = 0$$

$V_{TH} = IR$ and $V_{TH} = V_0 + V$

$$-I_0 + \frac{V_{TH} - V_0}{R} + \frac{V_{TH}}{R} = 0$$

$$V_{TH} = \frac{I_0 R + V_0}{2}$$



The equivalent circuit is