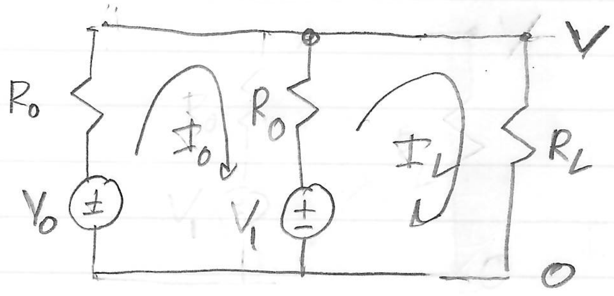
Physics 4L - Lecture 2



First we finish up our anaysis of circuits and conside KCL

① $-V_0 + R_0 I_0 + R_0(I_0 - I_L) + V_1 = 0$

OR

$$2R_0 I_0 - R_0 I_L = V_0 - V_1$$

② $-V_1 + R_0(I_L - I_0) + R_L I_L = 0$

or

$$-R_0 I_0 + (R_0 + R_L) I_L = V_1$$

$$\begin{pmatrix} 2R_0 & -R_0 \\ -R_0 & (R_0+R_L) \end{pmatrix} \begin{pmatrix} I_0 \\ I_L \end{pmatrix} = \begin{pmatrix} V_0 - V_1 \\ V_1 \end{pmatrix}$$

Recall $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$\begin{pmatrix} I_0 \\ I_L \end{pmatrix} = \frac{1}{R_0(R_0+2R_L)} \begin{pmatrix} R_0+R_L & R_0 \\ R_0 & 2R_0 \end{pmatrix} \begin{pmatrix} V_0 - V_1 \\ V_1 \end{pmatrix}$$

$$I_L = \frac{R_0(V_0 - V_1) + 2R_0 V_1}{R_0(R_0 + 2R_L)} = \frac{V_0 + V_1}{R_0 + 2R_L}$$

$$\quad = \left(\frac{V_0+V_1}{2}\right) \frac{1}{R_0/2 + R_L}$$

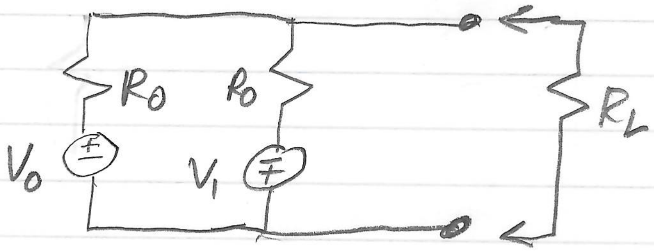
average voltage

Parallel internal resistances

The voltage across R_L is just

$$V_L = R_I I_L = \left(\frac{V_0 + V_1}{2}\right) \cdot \frac{R_L}{R_0/2 + R_L}$$

We can use this circuit to review Thevenin equivalence. Let's consider



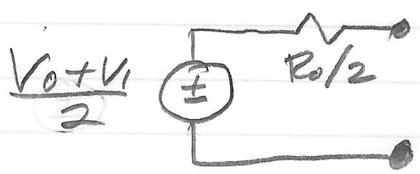
The Thevenin voltage is observed by taking $R_L \to \infty$. Then

$$V_{TH} = V_{open\,circuit} = V_L\,(R_L \to \infty)$$
$$= \frac{V_0 + V_1}{2}$$

The Thevenin current is observed by taking $R_L \to 0$. Then

$$I_{TH} = I_{short\,circuit} = I_L\,(R_L \to 0)$$
$$= \frac{V_0 + V_1}{2} \frac{1}{R_0/2}$$

$$R_{TH} = V_{TH} / I_{TH} = R_0/2$$

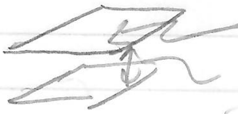


Time dependent components

We now consider the use of circuit elements that have a time dependent relation between current and voltage.

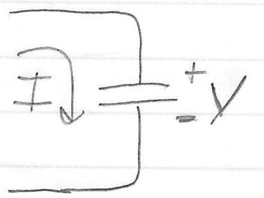
Capacitor $Q = C\ V$

Charge $= \int I\,dt$ — Geometry of confined electric field.



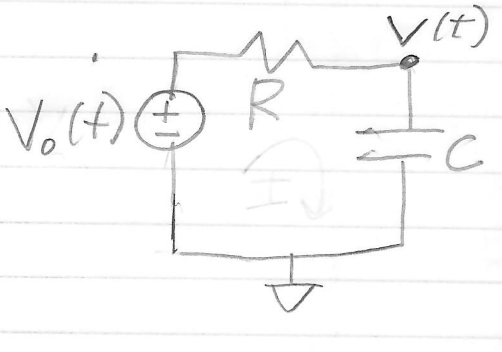
area A, gap d $\Big\}$ $C = \epsilon_0 \epsilon_r \dfrac{A}{d}$

ϵ_0: 8.9 F/m — ϵ_r: material, dielectric constant or ease of polarizability



$$V(t) = V(0) + \frac{1}{C}\underbrace{\int_0^t dt'\, I(t')}_{Q(t)}$$

We have $I(t) = C\dfrac{dV(t)}{dt}$



Kirchoff's Voltage Law

$$0 = \frac{V(t) - V_0(t)}{R} + C\frac{dV(t)}{dt}$$

$$(RC)\frac{dV(t)}{dt} + V(t) = V_0(t)$$

The product RC has units of time, the scale for charging the capacitor.

$$\tau \frac{dV(t)}{dt} + V(t) = V_0(t).$$

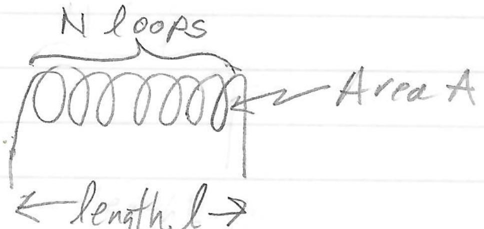
<u>Inductor</u>

$$\Phi_B = L\, I$$

Magnetic Flux $= \int \vec{B} \cdot \hat{n}\, d^2r$

Surface

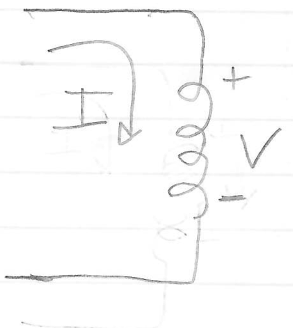
Geometry of confined magnetic field lines



$$L = \mu_0\, \mu_r \frac{N}{l} N A$$

$4\pi \times 10^{-7}$ H/m

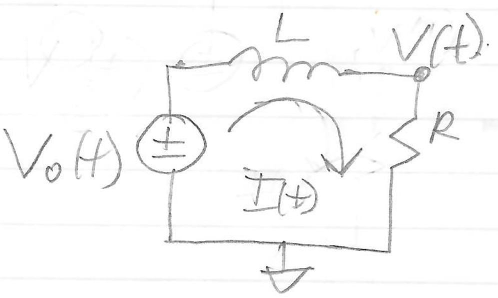
material in core, magnetic permeability or ease of polarizability



$$V(t) = L \frac{dI(t)}{dt}$$

$$I(t) = I_0(0) + \underbrace{\frac{1}{L}\int_0^{t'} dt'\, V(t')}_{}$$

$\Phi_B(t)$, recalling $\int \vec{E} \cdot \hat{n}\, dr = \frac{-\partial}{\partial t}\Phi_B$



$$0 = -V_0(t) + L\frac{dI(t)}{dt} + I(t)R$$

$$V(t) = I(t)\, R$$

$$\frac{L}{R}\frac{dV(t)}{dt} + V(t) = V_0(t)$$

like RC circuit but with a time constant $\tau = L/R$

In both cases, we need to solve the equation

$$\tau \frac{dV(t)}{dt} + V(t) = V_0(t)$$

for an arbitrary input $V_0(t)$.

What is the special role of "τ"?
If we consider the homogeneous equation

$$\tau \frac{dV(t)}{dt} + V(t) = 0$$

we have $\displaystyle\int_{V(0)}^{V(t)} \frac{dV'}{V'} = \frac{-1}{\tau}\int_0^t dt'$

$$\ln \frac{V(t)}{V(0)} = -t/\tau$$

$$V(t) = V_0 e^{-t/\tau}$$

τ sets the scale for time