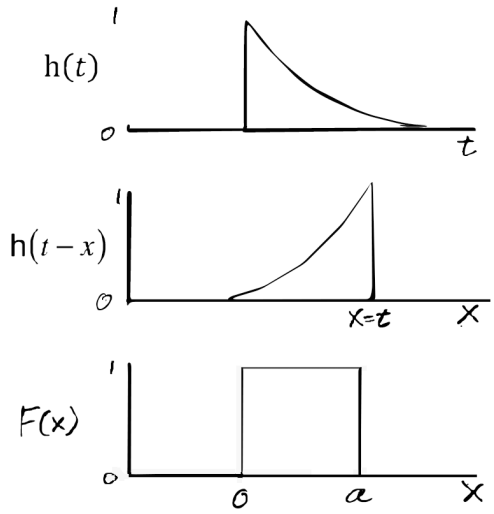
The convolution integral between the delta-function response of a circuit (the homogeneous equation), denoted h(t), and the external input, denoted F(t), is

$$\int_{-\infty}^{t} \mathrm{dx}\ \ \mathrm{h}(t-x)\ \mathrm{F}(x)$$

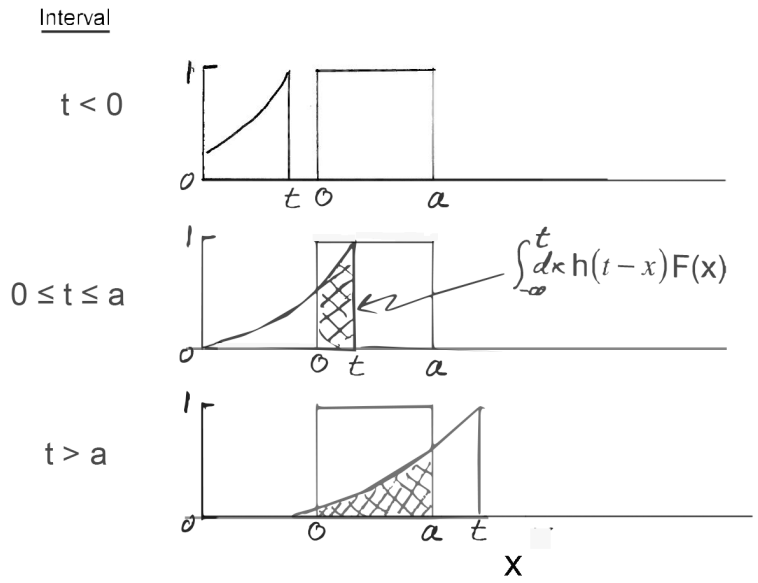
This can be seen as the overlap between h(-t), that is, h(t) running in reverse, and F(t). Here we evaluate the convolution for the choice of an exponential response

$$\mathrm{h}(t) = \frac{e^{t/\tau}}{\tau}$$

and a pulse of height 1 and width a, as shown below:



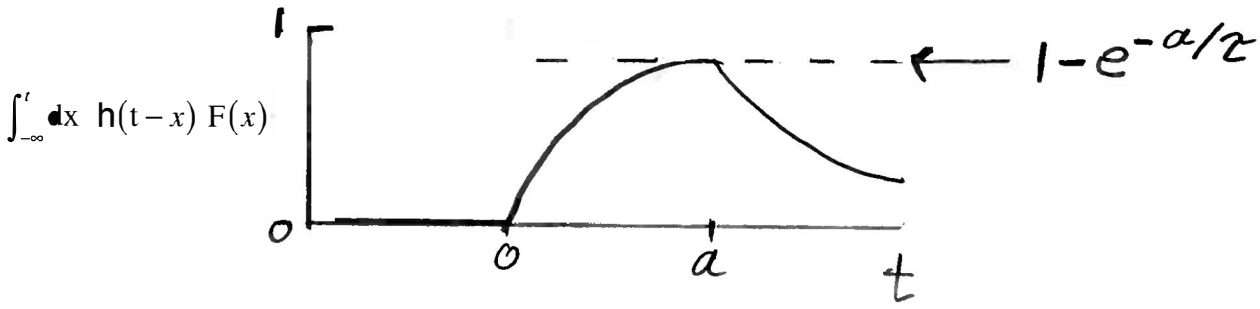

There are three intervals of integration. In pictures:



Analytically

:For t < 0, $\int_{-\infty}^{t} dx\ h(t-x)\ F(x) = 0$

For 0 ≤t ≤a, $\int_{-\infty}^{t} dx\ h(t-x)\ F(x) = \frac{1}{\tau}\int_{0}^{t} dx\ e^{-(t-x)/\tau} = \frac{\tau}{\tau}\, e^{-t/\tau}\left(e^{t/\tau} - 1\right) = 1 - e^{-t/\tau}$

For t > a, $\int_{-\infty}^{t} dx\ h(t-x)\ F(x) = \frac{1}{\tau}\int_{0}^{a} dx\ e^{-(t-x)/\tau} = \frac{\tau}{\tau}\, e^{-t/\tau}\left(e^{a/\tau} - 1\right) = e^{-(t-a)/\tau} - e^{-t/\tau}$

The result is an exponentially rising and decaying function that is specified by pieces; the function is continuous, but the derivative is not. Pictorially: