Supplemental data for

Ultra-slow oscillations in fMRI and resting-state connectivity: Neuronal and vascular contributions and technical confounds

Supplemental Text Box 1 - Relation of R^2 to $|\tilde{C}(f)|$

Consider a model that predicts a dependent variable from an independent variable. The goodness of fit of a model is usually described by a single parameter, R^2, which measures the portion of the variance in dependent variable that is predictable from the independent variable. For concreteness, let the dependent variable be the hemodynamic signal, denoted $H_{\mathrm{i}}(t)$, and the independent variable be the neurological signal, denoted $N_{\mathrm{i}}(t)$. The index i denotes the trial or epoch. Here we relate R^2 to the spectral coherence, $\tilde{C}(f)$, a common measure in signal analysis that reports the tendency of signals track each other within a frequency band centered at frequency f.

We consider a linear model for the filter $F(t)$ that transforms the neurological signal into a hemodynamic signal. The estimated hemodynamic signal is denoted $\hat{H}_{\mathrm{i}}(t)$ and is given by

$$\hat{H}_{\mathrm{i}}(t) = \int_0^t dx F(x) N_{\mathrm{i}}(t-x).$$

In the frequency domain, standard arguments lead to

$$\tilde{F}(f) = \frac{\left\langle \tilde{N}(f)\tilde{H}^{\star}(f) \right\rangle}{\left\langle |\tilde{N}(f)|^2 \right\rangle}$$

where $\langle \ldots \rangle$ refers to averaging over all trials and spectral estimates and $\tilde{N}_{\mathrm{i}}(f)$ and $\tilde{H}_{\mathrm{i}}(f)$ are the Fourier transform of the neurological and hemodynamic signals, respectively. The filter $F(t)$ is used to estimate $\hat{H}_{\mathrm{i}}(t)$ in R^2, i.e.,

$$R^2 \equiv 1 - \frac{\int dt \left\langle \left(H(t) - \hat{H}(t) \right)^2 \right\rangle}{\int dt \left\langle (H(t) - \langle H \rangle)^2 \right\rangle}.$$

This can be rewritten with the use of Parceval's theorem as

$$R^2 = 1 - \frac{\int df \left\langle \left| \tilde{H}(f) - \hat{\tilde{H}}(f) \right|^2 \right\rangle}{\int df \left\langle \left| \tilde{H}(f) \right|^2 \right\rangle} = \cdots = \frac{\int df \left| \tilde{C}(f) \right|^2 \left\langle \left| \tilde{H}(f) \right|^2 \right\rangle}{\int df \left\langle \left| \tilde{H}(f) \right|^2 \right\rangle}.$$

We see that R^2 is monotonic with respect to the integral of the squared coherence, $\left|\tilde{C}(f)\right|^2$, weighted by the power in the hemodynamic response. To the extent that R^2 is calculated for a band-limited signal centered at f_0 and extending over a frequency range of relatively constant power in $H(t)$, the weighting is constant and $R^2 \approx \left|\tilde{C}(f_0)\right|^2$.

Supplemental Table 1.

Variance explained between γ-band power and hemodynamic changes.

Species	**Correlated vascular quantity versus LFP**	**Neuronal-hemodynamic variance explained (R^2)**	**Reference**
Macaque	BOLD fMRI	~ 0.1	(Shmuel and Leopold, 2008)
Macaque	CBV fMRI (w/Mion)	~ 0.1	(Schölvinck et al., 2010)
Mice	Optical "BOLD"	~ 0.5*	(Ma et al., 2016)
Mice	Optical CBV	~ 0.1 - 0.2**	(Winder et al., 2017)
Mice	Arteriole diameter	~ 0.3*	(Mateo et al., 2017)

*R^2 taken as $|C(0.1\ Hz)|^2$; valid over a frequency range of relatively constant power in the hemodynamic transfer function. see **Supplemental Text Box 1**.
** Periods without movement